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Lagrangian and Eulerian velocity intermittency

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Abstract. Usual turbulence experiments, based on the Taylor hypothesis, differ from true Eulerian measurements. This is the origin of the apparent discrepancy between a recent two point correlation analysis and the multiplicative cascade picture. Indeed, both Eulerian and Lagrangian observations perfectly agree with this picture.

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In a recent work, Delour *et al.* [1] analyse the intermittency in several experimental records of turbulent velocity [2–4]. One of these experiments was of Lagrangian type, recording the velocity of a single particle following the flow [4]. The other experiments were of Eulerian type, looking at the velocity at a fixed point in the laboratory frame. One of the main results of their study is that the correlations deduced from the Lagrangian records are in good agreement with the multiplicative cascade picture invoked for modelizing the intermittency. On the opposite, Eulerian experiments seem to disagree with this cascade structure. In this paper, we argue that this apparent disagreement is due to the way the Eulerian experiments are made, through the Taylor frozen turbulence approximation [5], which cumulate spatial decorrelation and time decorrelation.

In order to stress the differences between Lagrangian and Eulerian records, Delour *et al.* [1] focussed on two quantities. One is the mean squared deviation of $\ln(|\delta v_\theta|)$: $C_2 = \langle(\delta\ln(|\delta v_\theta|))^2\rangle$, where $|\delta v_\theta|$ is the absolute value of the velocity increment on time intervals θ:

$$\delta v_\theta = v(t+\theta) - v(t);$$
$$\delta\ln(|\delta v_\theta|) = \ln(|\delta v_\theta|) - \langle\ln(|\delta v_\theta|)\rangle. \quad (1)$$

In the frame of the Taylor hypothesis, for Eulerian records, $v(t+\theta) - v(t)$ is interpreted as the longitudinal velocity difference $v_x(x) - v_x(x+\rho)$ where x is the direction of the average velocity $\mathbf{U}$ and $\rho = U\theta$.

The second quantity is the time correlation of the same variable $\delta l_\theta = \delta\ln(|\delta v_\theta|)$:

$$C_\theta(\tau) = \langle(\delta l_\theta(t+\tau))(\delta l_\theta(t))\rangle \quad (2)$$

where identity is assumed between the ensemble average $\langle * \rangle$ and the time average when t runs on the whole record.

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To be short, the results are as follows. First, Delour *et al.* [1] recognize that δv_θ can be considered as the product of two random independent variable, an amplitude a_θ and a sign s_θ, the latter being approximately Gaussian, of variance one, and short correlated in time. Then, defining:

$$\delta l_\theta = \delta\ln(a_\theta);$$
$$\delta\ln(|\delta v_\theta|) = \delta l_\theta + \delta\ln(|s_\theta|) \quad (3)$$

we have $C_2 = \langle(\delta l_\theta)^2\rangle + cst$, the constant being $\langle(\delta\ln(|s_\theta|))^2\rangle \simeq 1.24$. Moreover:

$$C_\theta(\tau) = \langle(\delta l_\theta(t+\tau))(\delta l_\theta(t))\rangle \quad (4)$$

for all τ larger than the correlation time of s_θ [6].

Delour *et al.* [1] find that, in agreement with the K.O.62 [5] theory, C_2 behaves logarithmically with θ, both for Eulerian ($(\mathrm{d}C_2/\mathrm{d}\ln\theta) = -\mu_E$), and Lagrangian ($(\mathrm{d}C_2/\mathrm{d}\ln\theta) = -\mu_L$) cases, up to a large scale correlation time T, above which C_2 is constant. C_2 is also constant under a dissipative scale θ_o. If not determined by the probe cut-off, $\theta_o = \eta/U$, where η is the Kolmogorov scale for the Eulerian case. In the Lagrangian case, $\theta_o = \tau_\eta = (\nu/\varepsilon)^{1/2}$, where ν is the kinematic viscosity, and ε the average dissipated power per unit mass. Physically, τ_η is the turn over time of the smallest eddies, of size η. μ_L is found much larger than μ_E ($\mu_E = 2.5\times10^{-2}$ and $\mu_L = 11\times10^{-2}$), in agreement with the Borgas and Sawford [7] theory.

As for $C_\theta(\tau)$, Delour *et al.* [1] note that, in agreement with the multifractal cascade models, it behaves logarithmically in the Lagrangian case within the range $\theta < \tau < T$ ($C_\theta(\tau) \simeq -\mu_L\ln(\tau/T)$). However, the behaviour is quadratic in $\ln(\tau)$ for the Eulerian case ($C_\theta(\tau) \propto (\ln(\tau/T))^2$). We shall show below how these apparently contrasting results can be reconciliated.

For the Lagrangian case, we keep the same definition of the correlation, and take as granted that:

$$^{L}C_{\theta}(\tau) = \langle(\delta l_{\theta}(t+\tau))(\delta l_{\theta}(t))\rangle = \mu_L \ln(T/\tau). \quad (5)$$

For the Eulerian case, we define:

$$^{E}C_{\rho}(r) = \langle(\delta l_{\rho}(x+r,t))(\delta l_{\rho}(x,t))\rangle \quad (6)$$

with $\delta l_{\rho}(x) = \ln(a_{\rho}(x)) - \langle\ln(a_{\rho})\rangle$. We claim that it differs from the above measured quantity, based on the Taylor Hypothesis, which we shall call $^{Ta}C_{\rho}(\tau)$:

$$^{Ta}C_{\rho}(\tau) = \langle(\delta l_{\rho}(x,t+\tau))(\delta l_{\rho}(x,t))\rangle \quad (7)$$

with $r = U\tau$ and $\rho = U\theta$.

According to the cascade models, we should have $^{E}C_{\rho}(r) = \mu_E \ln(L/r)$, where $L \simeq U'T$ (U' is the velocity rms). This can be written:

$$\delta l_{\rho}(x,t+\tau) = \alpha_E \delta l_{\rho}(x+r,t+\tau) + \delta \quad (8)$$

where $\alpha_E = {}^{E}C_{\rho}(r)/\langle\delta l_{\rho}^2\rangle$ and δ is decorrelated from $\delta l_{\rho}(x+r,t+\tau)$.

If we remark that the fluid element, being at x at time t, is at $x+r$ at time $t+\tau$, we can write, in the same way:

$$\delta l_{\rho}(x+r,t+\tau) = \alpha_L \delta l_{\rho}(x,t) + \delta' \quad (9)$$

with $\alpha_L = {}^{L}C_{\theta}(\tau)/\langle\delta l_{\theta}^2\rangle$.

Then:

$$\delta l_{\rho}(x,t+\tau) = \alpha_E \alpha_L \delta l_{\rho}(x,t) + \alpha_E \delta' + \delta. \quad (10)$$

We assume that δ, which is decorrelated from $\delta l_{\rho}(x+r,t+\tau)$ is also decorrelated from $\delta l_{\rho}(x,t)$, as δ' is. This is justified by the great number of degrees of freedom for $\delta l_{\rho}(x,t)$. Then we have:

$$^{Ta}C_{\rho}(\tau) = \alpha_L \alpha_E \langle\delta l_{\rho}^2\rangle = (\mu_L \mu_E/\langle\delta l_{\theta}^2\rangle)\ln(T/\tau)\ln(L/r). \quad (11)$$

Put in other way, the correlation is proportional to the probability to δl_{ρ} not having been decorrelated, either in space or in time.

As quoted above, an important time scale is the Lagrangian dissipative time $\tau_{\eta} = (\nu/\varepsilon)^{1/2}$. Using the relation: $\varepsilon = 15\nu U'^2/\lambda^2$, where λ is the Taylor scale and U' the velocity rms, we have $\tau_{\eta} = \lambda/U'\sqrt{15} \simeq \lambda/4U'$.

In general θ is chosen smaller than τ_{η}. Then $\langle\delta l_{\theta}^2\rangle = \mu_L \ln(T/\tau_{\eta}) \simeq \mu_L \ln(R_{\lambda})$ where R_{λ} is the Taylor scale based Reynolds number. The behaviour of $C_{\rho}(\tau)$ will be different, depending if τ is larger or smaller than τ_{η}. $\tau = \tau_{\eta}$ corresponds to $r = \lambda U/4U'$.

If $\tau > \tau_{\eta}$ ($r > \lambda U/4U'$), approximating $\ln(T/\tau)\ln(L/r) \simeq \ln(T/\tau)^2$, we have:

$$C_{\rho}(\tau) = \mu_E \ln(T/\tau)^2/\ln(R_{\lambda}) \quad (12)$$

in full agreement with Delour *et al.*[1].

If $\tau < \tau_{\eta}$ ($r < \lambda U/4U'$), there is no time decorrelation, and:

$$C_{\rho}(\tau) = \mu_E \ln(L/r) \quad (13)$$

To summarize, the analysis by Delour *et al.* [1] of Eulerian experiments is in good agreement with the multiplicative cascade structure if account is taken of the deviations from the Taylor hypothesis. The corresponding time decorrelation has no effect for scales smaller than $\lambda U/4U'$, in agreement with their observations. There is no contradiction between Lagrangian and Eulerian observations, and, as they propose, the logarithmic correlation seems to capture all the physics of intermittency.

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References

1. J. Delour, J.F. Muzy, A. Arneodo, Eur. Phys. J. B **23**, 243 (2001); J. Delour, Ph.D. thesis, Université de Bordeaux, (2001) unpublished
2. Y. Malecot, C. Auriault, H. Kahalerras, Y. Gagne, O. Chanal, B. Chabaud, B. Castaing, Eur. Phys. J. B **16**, 549 (2000)
3. O. Chanal, B. Chabaud, B. Castaing, B. Hébral, Eur. Phys. J. B **17**, 309 (2000)
4. N. Mordant, P. Metz, O. Michel, J.F. Pinton, Phys. Rev. Lett. **87**, 214501 (2001); N. Mordant, Ph.D. thesis École Normale Supérieure de Lyon (2001), unpublished
5. A.S. Monin, A.M. Yaglom, *Statistical fluid dynamics: Mechanics of turbulence* (MIT Press, 1987)
6. I am indebted to Laurent Chevillard for help in clarifying this point
7. M.S. Borgas, B.L. Sawford, Phys. Fluids **6**, 618 (1994)